An Investigation of Correlation Region in Maneuvering Multi-Target Tracking

Hongren Zhou* and K.S.P. Kumar†
University of Minnesota, Minneapolis, Minnesota

The statistical properties of the norm of the innovation vector, the size of the correlation region, and the probability of accepting a correct return in Maneuvering Multi-Target Tracking are investigated by means of Monte Carlo simulation with three different state models, different maneuvering accelerations of targets, and the variance of state noise. Investigation shows that the state model of maneuvering targets and the corresponding adaptive algorithm suggested previously by Zhou and Kumar have the best capability of those examined to adapt to maneuvering of targets and give relatively stable correlation region and the probability of accepting a correct return.

I. Introduction

IT is well known that the data correlation process that determines which sensor observations, if any, are to be used in the track update by the tracking filter is one of the basic problems of tracking in a multitarget environment. The first problem that one faces in the data correlation process is the determination of the size of the correlation region (or gate, window, etc.) in which data association, or correlation, will be carried out.

In the case of nonmaneuvering targets, the size of correlation region is a constant, and a number of methods for multi-target tracking (without maneuvering have been developed.[1,2] However, in the maneuvering multi-target tracking (MMTT) case, the question of whether the size of correlation region should be adaptable so that the maneuvering of targets can be followed arises. R.E. Lefferts[3] examined the design of correlation region for a track-while-scan system and concluded that for a maneuvering target the size of the correlation region must be different from the nonmaneuvering case and readjusted to maintain a constant probability of successful correlation.

This paper deals with the problem of the correlation region in the case of one site, maneuvering multi-target. A contrary conclusion to that of Lefferts is obtained if the state model of maneuvering targets and the corresponding adaptive algorithm suggested by Zhou and Kumar[4] is used.

Usually, the norm of the innovation vector normalized by the inverse of its covariance is used to form a test to decide whether an observation should be retained or discarded. Hence the statistical properties of this norm are of major concern, especially when the state noise is assumed to be non-Gaussian. Based on the statistical properties, the behavior of the size of the correlation region and the probability of accepting a correct return can be examined. This paper investigates these problems from the following three aspects:

1) The state model of maneuvering target. Three different models are taken into consideration: Zhou and Kumar's,[4] Singer's,[5] and two-state (position and velocity) model;[2]

2) The covariance of state noise. From the viewpoint of design, this is an important and controllable parameter in the Kalman Filtering Equation;

3) The maneuvering accelerations of targets. Since the innovation vector can be used to detect the maneuver of a target,[8] the change of the norm of the innovation vector with the maneuvering acceleration is of importance.

Monte Carlo simulation is used in this paper. The statistical properties of the norm of the innovation vector are described in Sec. II. The size of the correlation region and the probability of accepting a correct return are discussed in Sec. III. Finally, some conclusions are drawn in Sec. IV.

II. Statistical Properties of the Norm of the Innovation Vector

Let the discretized equations of motion for a target being tracked be modeled by

$$X(k+1) = \Phi(k)X(k) + G(k)U(k) \tag{1}$$

where $X(k)$ is the $n\times 1$ state vector of the tracked target, $\Phi(k)$ is the $n\times n$ transition matrix, and $U(k)$ is the $p\times 1$ state excitation vector, which is assumed to be either white Gaussian or non-Gaussian with either zero mean or nonzero mean and covariance $Q(k)$. The particular set of assumptions that are used depends on which of the three state models, referred to in the previous section, is employed.

Let the observation equation of the sensor used to track targets be as follows:

$$Y(k) = H(k)X(k) + V(k) \tag{2}$$

where $Y(k)$ is the $m\times 1$ sensor measurement vector and $V(k)$ is an $m\times 1$ white Gaussian observation noise vector with zero mean and covariance $R(k)$.

In multi-target tracking, the innovation vector $[Y(k)-H\hat{X}(k|k-1)]$ obtained from the Kalman filter is normalized by the inverse of its covariance and is used to form a norm,

$$\begin{aligned} g(k) &\triangleq \|Y(k)-H\hat{X}(k|k-1)\|^2 \\ &\triangleq [Y(k)-H\hat{X}(k|k-1)]'V^{-1}(k) \\ &\quad\times[Y(k)-H\hat{X}(k|k-1)] \end{aligned} \tag{3}$$

where $\hat{X}(k|k-1)$ is the one-step-predicted state vector,

$$V(k) = HP(k|k-1)H' + R(k) \tag{4}$$

is the covariance of the innovation vector and $P(k|k-1)$ is the covariance of the one-step-predicted error $\tilde{X}(k|k-1) = X(k)-\hat{X}(k|k-1)$.

The test to determine whether or not a measurement $Y(k)$ is a candidate return from the target tracked consists of examining whether or not the norm of the innovation vector is

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*Graduate Student, Department of Electrical Engineering.

†Professor, Department of Electrical Engineering.

less than a threshold γ, i.e.,

$$g(k) \leq \gamma \tag{5}$$

where γ is also called the size of the correlation region. Obviously, the value of γ is related not only to the probability of accepting a correct return but also to the probability of false alarm.

Some questions that arise in the MMTT case are 1) how does $g(k)$ change when the target being tracked is maneuvering, and 2) what should one do if maintaining the probability of accepting or rejecting the correct return constant is desired? To answer these questions, the statistical properties of $g(k)$ and the influence of a variety of factors, such as models of maneuvering target and filtering algorithm, maneuvering accelerations of the target, the main design parameters of the system, etc., on the statistical properties of $g(k)$ must be investigated. Three different models are considered in this paper.

Model I is suggested by Zhou and Kumar.[4] The main characteristic of this model is that the "current" probability density function of target maneuvering acceleration is described by a modified Rayleigh density with variable mean-value from which an adaptive filtering algorithm for the mean and variance of the maneuvering acceleration was developed. It has been shown that this model can estimate the target state well in both maneuvering and nonmaneuvering cases.

Model II is suggested by Singer,[5] in which the maneuvering acceleration of the target is assumed to have an approximately uniform density function in the interval $[-a_{\max}, a_{\max}]$ where $a_{\max}$ is the maximum maneuvering acceleration and a standard Kalman filtering algorithm is used.

Model III is a two-state model, with only position and velocity of target involved, which has been used by several authors[2,6,7] for nonmaneuvering multi-target tracking.

Probability Density of Norm $g(k)$

It is well known[9] that the innovation vector is also Gaussian if both the state noise and the observation noise are Gaussian. Thus $g(k)$ will have an m degrees-of-freedom chi-square distribution.[10] Unfortunately, the state noises in Models I and II are both assumed to be non-Gaussian. Hence the density functions of $g(k)$ for these two models are to be determined.

Figures 1a and 1b give the samples of probability density of $g(k)$ for Models I and II, respectively. The numerical values in the simulation are $m=1$, the variance of the state noise $\sigma_s^2 = 400$ for Models I and II, and 3820 for Model III. The variance of the observation noise is

$$R(k) = \left(\eta X(k) + \Delta X_0\right)^2 \cdot E\left[w^2(k)\right] \tag{6}$$

where η is a relative error coefficient, ΔX_0 is a fixed measuring error, $w(k)$ is a pseudorandom number which is normal with zero mean and variance $\sigma^2 = 1$.

Figure 1 shows that the densities for Models I and II are both close to chi-square distribution. The density function of one-degree-of-freedom chi-square distribution is[10]

$$p(x) = \frac{1}{\sqrt{2\pi x}} e^{-x/2}, \qquad x > 0 \tag{7}$$

and the mean value and variance of x are equal to 1 and 2, respectively. Assume that $g(k)$ obeys a chi-square distribution, but with different mean and variance from those of Eq. (7). Let

$$g(k) = \alpha x + \beta \tag{8}$$

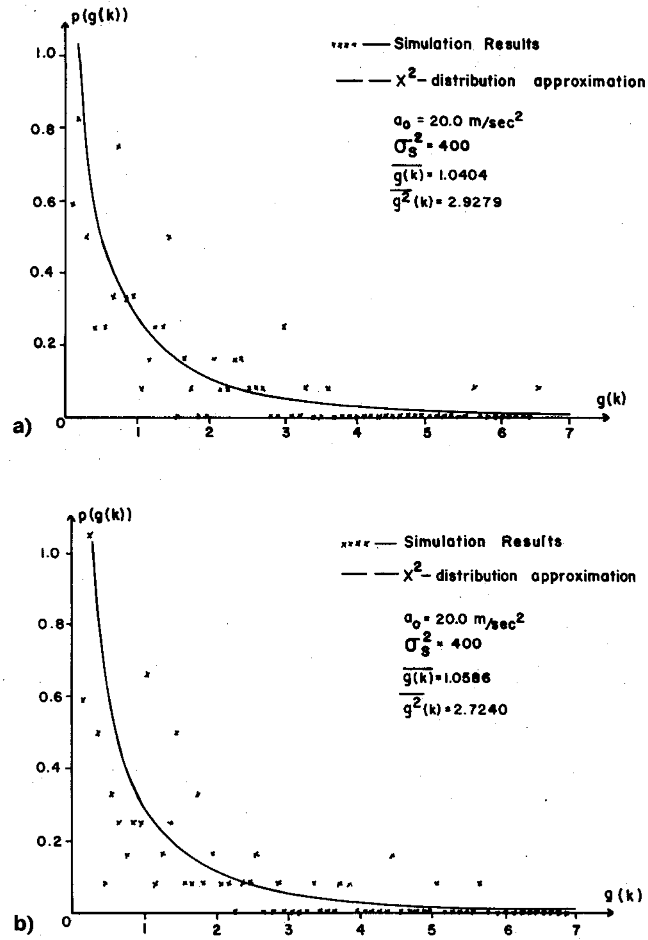

Fig. 1 Samples of density of norm $g(k)$ of innovation vector: a) Model I; b) Model II.

Then it is easy to find

$$\alpha = \sqrt{\tfrac{1}{2}\left(\overline{g^2(k)} - \overline{g(k)}^{\,2}\right)} \tag{9}$$

$$\beta = \overline{g(k)} - \sqrt{\tfrac{1}{2}\left(\overline{g^2(k)} - \overline{g(k)}^{\,2}\right)} \tag{10}$$

where $\overline{g(k)}$ and $\overline{g^2(k)}$ are the mean and the mean squared value of $g(k)$. Thus the density function of $g(k)$ is

$$p(g(k)) = \frac{1}{\alpha} \cdot \frac{1}{\sqrt{(2\pi/\alpha)(g(k)-\beta)}} e^{-g(k)-\beta/2\alpha}, \qquad g(k) > \beta \tag{11}$$

The density functions of $g(k)$ approximated by the equation above for Models I and II are shown in Fig. 1 as well.

Mean and Variance of $g(k)$

The changes of the mean $\overline{g(k)}$ and the mean squared value $\overline{g^2(k)}$ of $g(k)$ with different models, maneuvering accelerations a_0 of targets and the variance σ_s^2 of the state noise are investigated.

Figure 2 gives the relationship of $\overline{g(k)}$ and $\overline{g^2(k)}$ vs a_0 and σ_s^2 for Model I. The mean $\overline{g(k)}$ is very close to 1 and the mean squared value $\overline{g^2(k)}$ is a little larger than 2. Both $\overline{g(k)}$ and $\overline{g^2(k)}$ do not change much, even though a_0 and σ_s^2 vary over a large range. Taking $\sigma_s^2 = 400$ as an example, the change

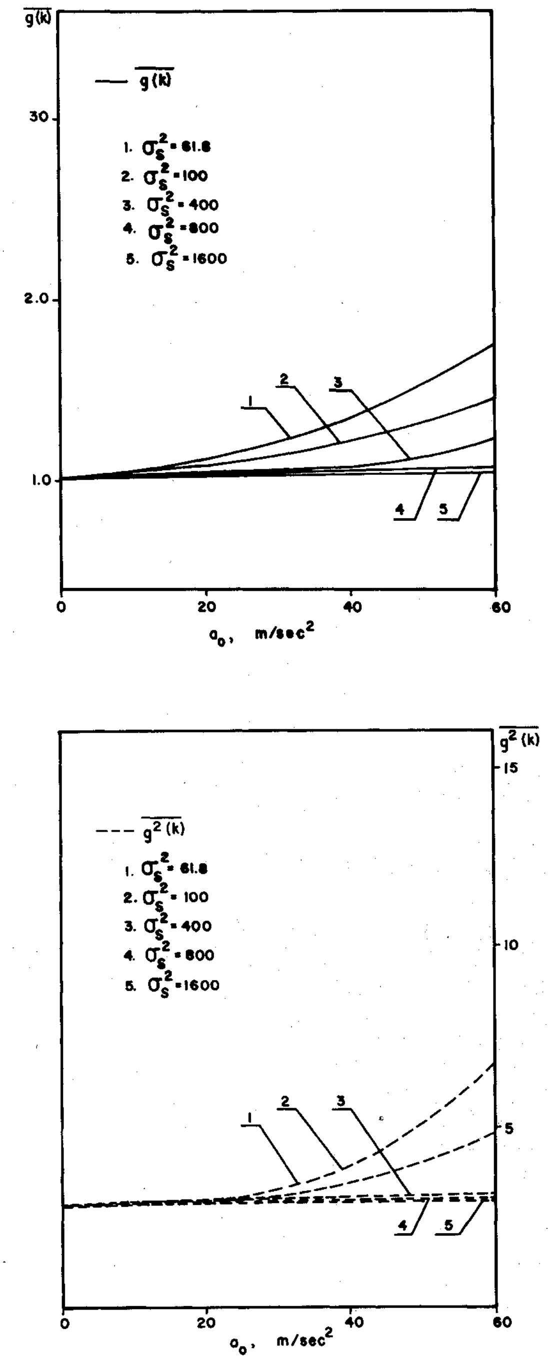

Fig. 2 $\overline{g(k)}$ and $\overline{g^2(k)}$ vs a_0 and σ_s^2 for Model I.

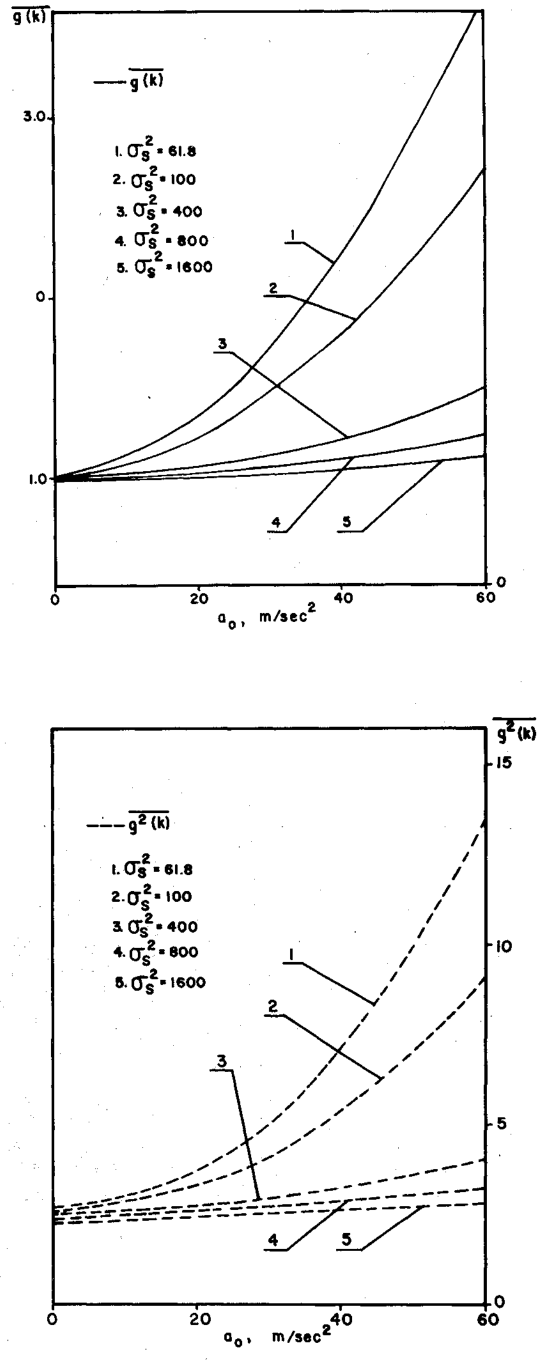

Fig. 3 $\overline{g(k)}$ and $\overline{g^2(k)}$ vs a_0 and σ_s^2 for Model II.

of $\overline{g(k)}$ is about 10% and the change of $\overline{g^2(k)}$ is less than 11.4% in the whole region of changes of a_0.

Figure 3 shows that the changes of $\overline{g(k)}$ and $\overline{g^2(k)}$ for Model II with the changes of a_0 and σ_s^2 are obviously larger than those in Model I. In the case of $\sigma_s^2 = 400$, $\overline{g(k)}$ and $\overline{g^2(k)}$ change 48% and 60.5%, respectively, while the acceleration of target varies from 0 to 60 m/s^2. These are because of the poor capability of this model to track maneuvering acceleration and the innovation $[Y(k) - H\hat{X}(k|k-1)]$ becomes larger as the acceleration of target increases.

Figure 4 presents the changes of $\overline{g(k)}$ and $\overline{g^2(k)}$ with a_0 and σ_s^2 for Model III. It can be seen that $\overline{g(k)}$ and $\overline{g^2(k)}$, for example, change 99.8% and 147.5%, respectively, if $\sigma_s^2 = 3820$ and a_0 varies from 0 to 60 m/s^2. This means that this model has no capability for tracking maneuvering targets.

Figures 2 to 4 show that Model I has the most stable $\overline{g(k)}$ and $\overline{g^2(k)}$ among the three models. This is because Model I has the best capability to track maneuvering targets.

III. Size of the Correlation Region and the Probability of Accepting the Correct Return

Once the threshold γ or the size of correlation region is given, the probability that the target being tracked falls into the correlation region, i.e., that the MMTT system accepts the correct return, is

$$P_a(g(k) \leq \gamma) = \int_\beta^\gamma P(g(k))\, d(g(k))$$

$$= \int_0^{(\gamma-\beta)/\alpha} \frac{1}{\sqrt{2\pi z}} e^{-z/2}\, dz \qquad (12)$$

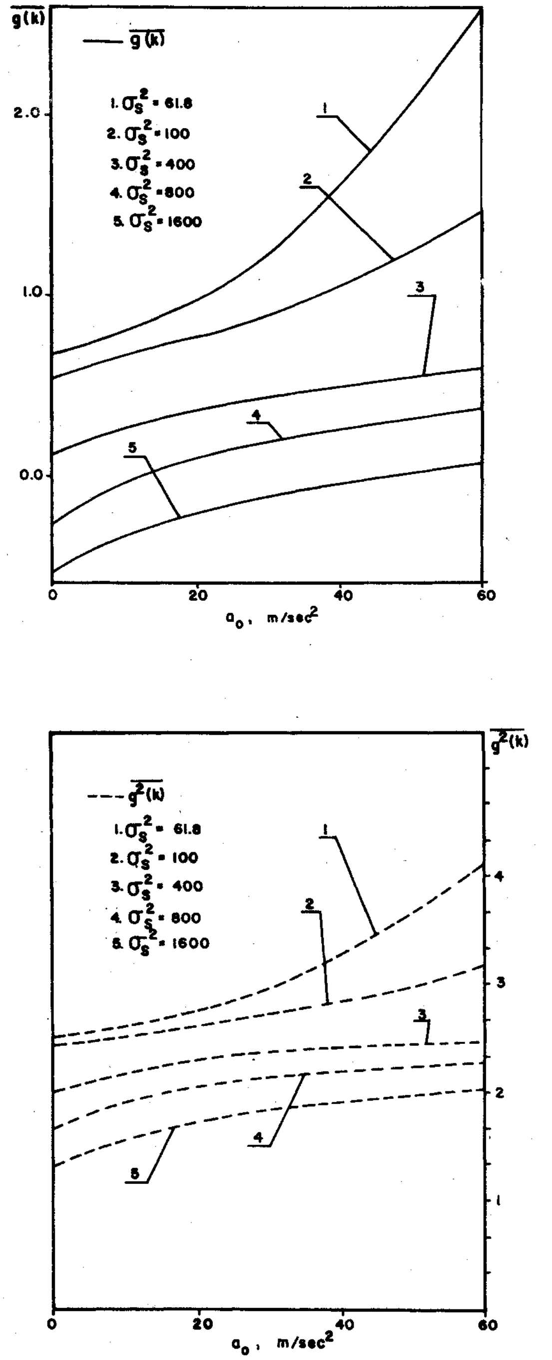

Fig. 4 $\overline{g(k)}$ and $\overline{g^2(k)}$ vs a_0 and σ_s^2 for Model III.

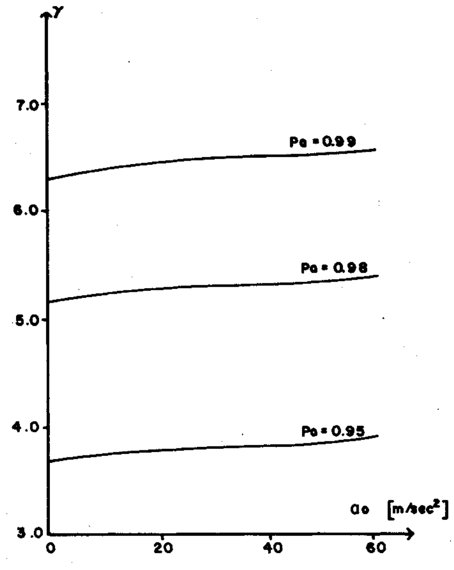

Fig. 5 γ vs a_0 under given P_a for Model I.

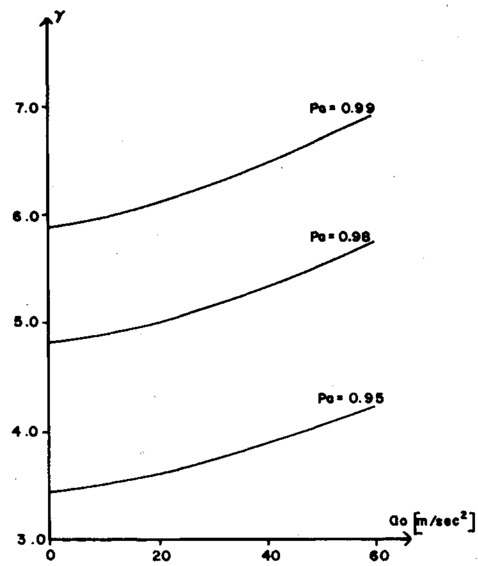

Fig. 6 γ vs a_0 under given P_a for Model II.

where α and β are given in Eqs. (9) and (10). The probability of rejecting the correct return is

$$P_r(g(k) > \gamma) = 1 - P_a \tag{13}$$

Obviously the larger γ is, the smaller the probability of rejecting the correct return. But the price paid is a large false alarm probability and a heavier burden of processing the data coming from clutter or other targets not in this track. Hence, γ and P_a or P_r are the important performance indices in MMTT system. In this section, the influences of modeling maneuvering targets, maneuvering acceleration of target and the variance of the state noise on γ and P_a are investigated.

Probability P_a Given

In an MMTT system, suppose one wants to keep the probability of accepting the correct return as a constant, then what should γ be under different maneuvering acceleration a_0 and the variance σ_s^2 of state noise?

Figure 5 answers this question for Model I where $P_a = 0.99$, 0.98, and 0.95. It is apparent that the curves γ vs a_0 are very flat. The changes of γ for $P_a = 0.99$, 0.98, and 0.95 are less than 2.6, 3.9, and 4.5%, respectively, when a_0 varies from 0 to 60 m/s^2. Hence it can be said that the values of γ are stable when the target being tracked is maneuvering.

Figure 6 shows the results for Model II. Obviously, the changes of the size of the correlation region demanded are much larger than those in Model I. For $P_a = 0.99$, 0.98, and 0.85, γ changes 14.5, 15.5, and 17.6%, respectively, as a_0 increases from 0 to 60 m/s^2. Therefore, if one uses this model in the MMTT case, it appears necessary to find a method to adjust the sizes γ so that a constant probability of accepting the correct return can be maintained.

Both the results in Figs. 5 and 6 are obtained under the numerical value of $\sigma_s^2 = 400$.

For Model III, the size of correlation region changes 26.0, 27.6, and 30.7% corresponding to the probability $P_a = 0.99$, 0.98, and 0.95, respectively, when a_0 varies from 0 to 60 m/s^2

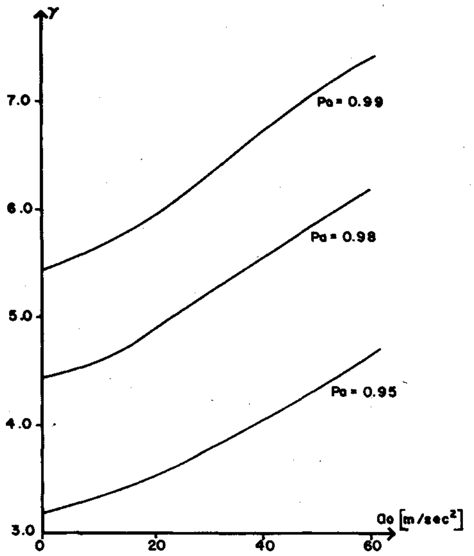

Fig. 7 γ vs a_0 under given P_a for Model III.

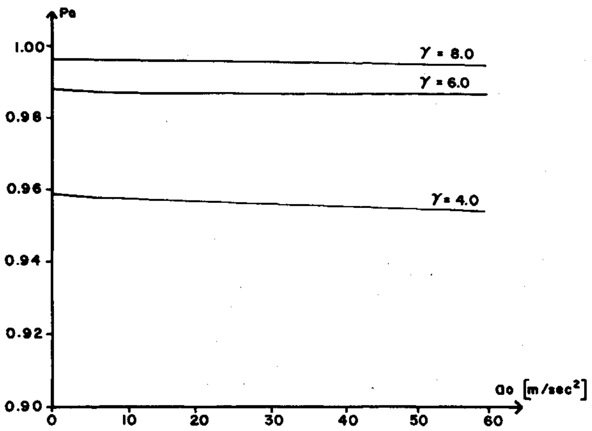

Fig. 8 P_a vs a_0 under given γ for Model I.

(Fig. 7). It seems that this model is not suitable to the MMTT case.

Size of the Correlation Region Given

Figure 8 presents the changes of the probability of accepting the correct return P_a with maneuvering acceleration of targets under different given sizes of correlation region for Model I. For $\gamma = 8$, 6, and 4, the changes of the probability P_a are only 0.06, 0.18, and 0.51%, respectively. This shows once again that Model I can adapt to maneuvering of targets.

Figure 9 shows the results for Model II. The probability P_a decreases as the maneuvering acceleration a_0 increases. P_a can change 0.26, 0.8, and 2.3% for $\gamma = 8$, 6, and 4. The larger the maneuvering acceleration is, the lower the probability of accepting the correct return.

Figure 10 exhibits drastic changes of P_a for Model III. As the maneuvering acceleration of target increases, the probability P_a decreases. For $\gamma = 8$, 6, and 4, it changes by 0.53, 1.5, and 4.7%, respectively.

IV. Conclusions

The investigation of this paper has led to the following conclusions:

For Models I and II considered in this paper and characterized, respectively, by a Rayleigh and a uniform probability density function for target maneuvering acceleration, the

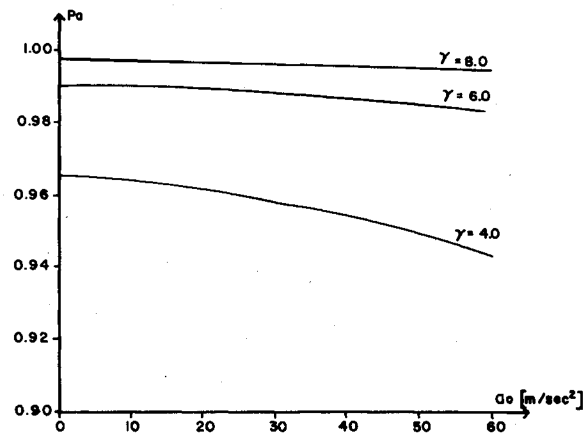

Fig. 9 P_a vs a_0 under given γ for Model II.

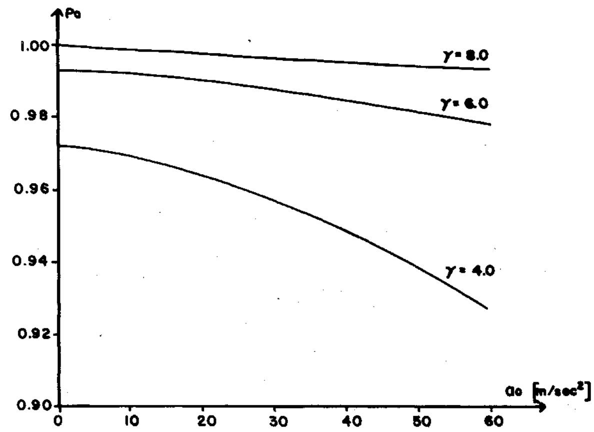

Fig. 10 P_a vs a_0 under given γ for Model III.

norm of the innovation vector can be approximated to have chi-square distribution, but the mean and the variance vary with the design parameters.

Model I can adapt itself to target maneuvers. The mean and the mean squared value of the norm of the innovation vector in this model have very good relative stability when the target maneuvers over a wide range. The probability of accepting the correct return and the size of the correlation region do not change drastically with the changes of the variance of state noise and of maneuvering accelerations of targets. Therefore, it is not necessary to take any other measures to maintain the probability that correct correlation and the size of the correlation region will be constant. In this sense, Model I is very suitable for use in Maneuvering Multi-Target Tracking.

From the viewpoint of system design, the larger variance σ_s^2 of state noise will make the size of the correlation region and the probability of correct correlation more stable with respect to maneuvering of targets. It also makes the covariance of the tracking error larger, however. Hence a compromise must be made in system design.

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